Superconducting Qubits

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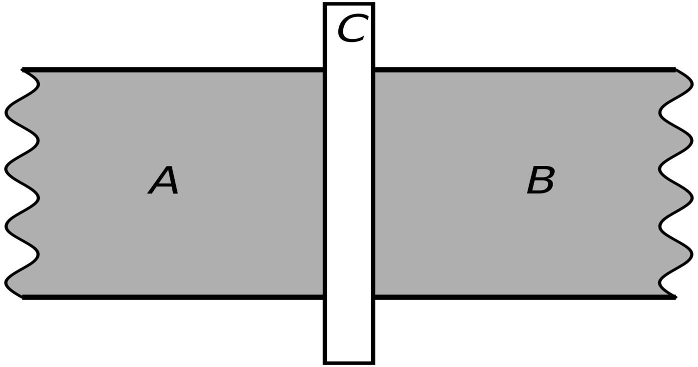
Introduction

Superconducting qubits are basically **Josephson Junctions**(JJ), whose working principle is the **Josephson Effect**. Proposed in 1963 by Brian Josephson, for which he got the Noble Prize in 1973 following experimental verification.

A JJ is a Superconductor - Insulator - Superconductor junction

The Josephson Effect refers to a phenomenon in which a **supercurrent** passes through the insulator when a potential difference is applied across the superconductors of a JJ.

The JJ is a non-linear circuit element,
and is also the only known dissipation-less non-linear circuit element.

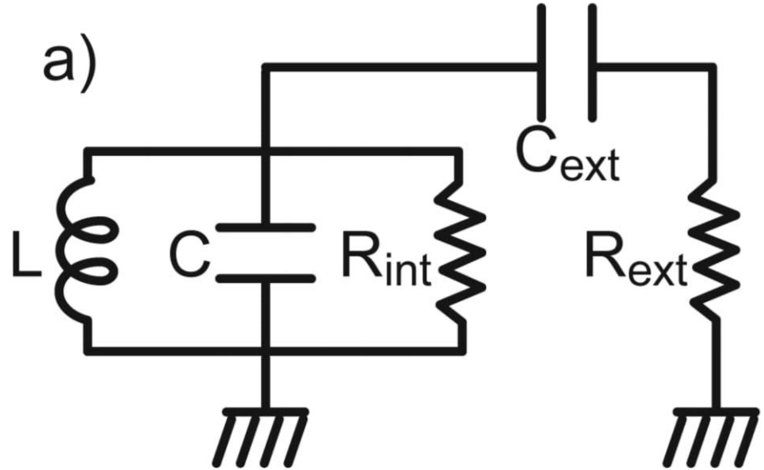


Linear Superconducting LC circuit

A JJ can be modeled as a non-linear quantum LC circuit.

In order to understand what non-linearity really means in the context of a quantum circuit, it is insightful to first understand how a simpler **linear** quantum LC circuit works.

Figure a) shows a lossy LC oscillator, which has both internal and external losses.



Linear Superconducting LC circuit

It has a hamiltonian given by -

$$H_{LC} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Introducing ladder operators similar to the Quantum Harmonic Oscillator -

$$\hat{Q} = i\sqrt{\frac{\hbar}{2Z_o}}(\hat{a}^\dagger - \hat{a})$$

$$\hat{\Phi} = \sqrt{\frac{\hbar Z_o}{2}}(\hat{a}^\dagger + \hat{a})$$

$$\Rightarrow H_{LC} = \hbar\omega_o \hat{a}^\dagger \hat{a}$$

Where we have introduced a characteristic impedance and angular frequency given by -

$$Z_o = \sqrt{\frac{L}{C}}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Linear in $\hat{a}^\dagger \hat{a}$

Linear Superconducting LC circuit

So in this context, we have a **constant energy spacing** between the levels and the notion of well defined energy levels reduces to - $\hbar\omega_o >> k_B T$

Lifetime $T_1 = RC$

Where **R** is the impedance of the circuit given by -

$$\frac{1}{R(\omega)} = |j\omega C + \frac{1}{j\omega L} + \frac{1}{R_{int}} + \frac{j\omega C}{1+j\omega CR}|$$

Linear Superconducting LC circuit

With standard microfabrication techniques, we can have L and C values as low as L ~ 0.1 nH and C ~ 1 pF. This gives a frequency in the microwave region -

$$\frac{\omega_o}{2\pi} \sim 16\, GHz$$

Given that $1\, GHz * h/k_B \sim 50\, mK$

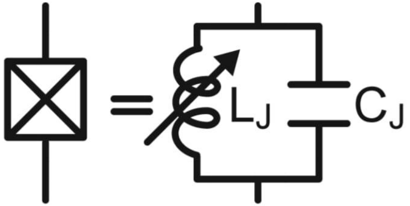
the condition of well defined energy levels $\hbar\omega_o >> k_B T$

can be easily satified by operating in a 20 mK dilution refrigerator.

And minimal losses can be achieved by using superconductors, typically **Al or Nb** decoupled from the environment.

Why do we need a Josephson Junction?

b)



1. While we could model a linear LC circuit as having discrete energy levels, establishing quantum behaviour experimentally is hard.

2. We ultimately need a qubit, which is a 2 level quantum system. Hence it would be favourable to have a non-uniform energy spacing where the gap becomes very large between the 2nd and 3rd excited states.

Ion Trap Qubits Vs. Superconducting Qubits

Ion Trap Qubit properties:

$$T_1 \sim a\, few\, years$$

$$T_2 \sim 10\, seconds$$

$$T_\pi \sim 5\, \mu\, s$$

$$Low\, error\, per\, gate \sim\ 0.48\%$$

Superconducting Qubit properties:

$$T_1 \sim 100\, microseconds$$

$$T_2 \sim 100\, microseconds$$

$$T_\pi \sim 4 - 20\, ns$$

$$Low\, error\, per\, gate \sim\ 0.2\%$$

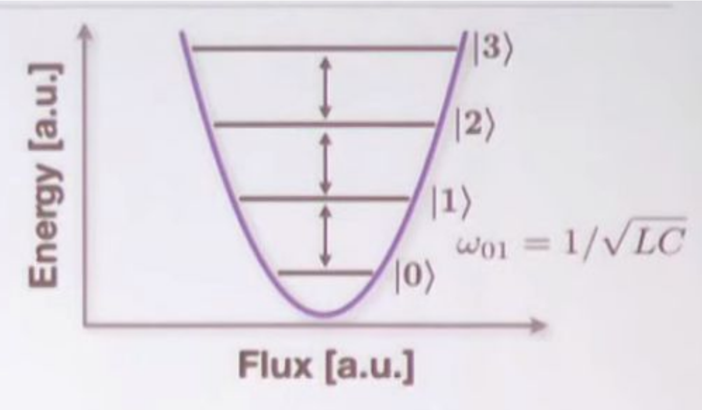
Why should we use superconducting qubits?

-If we want to “artificial atoms” which can be realized in the solid state, then superconducting structures are the way to go, due to no dissipation(radiation) of energy.

Artificial atoms

With standard microfabrication techniques values of $L \sim 0.1\, nH$ and $C \sim 1\, pF$ can be obtained leading to frequencies in the microwave regime or $\omega_{01} = \frac{1}{\sqrt{LC}} \sim 10GHz \sim 0.5K$.

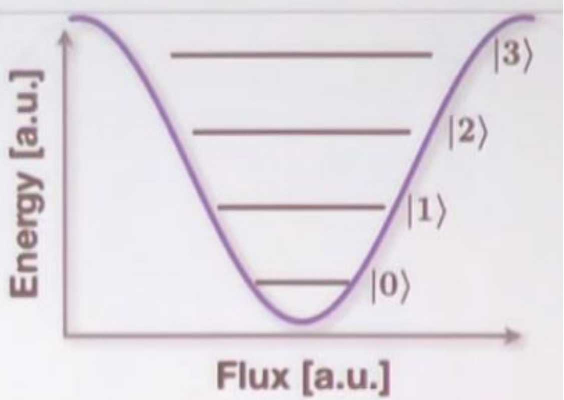
We can use a microwave source to excite the states, which is similar to irradiating atoms with light.



But as the energy separation is the same everywhere, the system can also move to higher states and this is a problem, cause then we no longer have a well behaved 2 qubit system. **So, how to avoid this pitfall?**

$$I = I_0 sin(\frac{2\pi\phi}{\phi_0})$$

Non-linearity!



We can introduce non-linearity using Josephson junctions.

-What is the Hamiltonian and what kind of energy spectrum do we obtain?

And as you can see the nonlinearity leads to a non uniform spacing of the energy levels which is good.

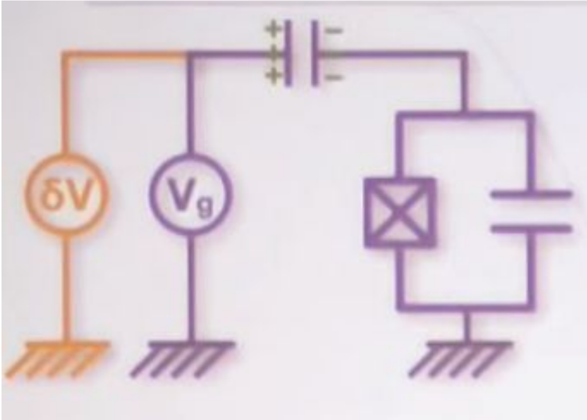
$$L_J(\phi) = (\frac{\partial I}{\partial \phi})^{-1} = \frac{\phi_0}{2\pi I_0}\frac{1}{cos(\frac{2\pi\phi}{\phi_0})}$$

$$E = \int dt V(t) I(t) = \int dt (\frac{d\phi}{dt})(I_0 sin\frac{2\pi\phi}{\phi_0}) = -E_J cos(\frac{2\pi\phi}{\phi_0})$$

E_J Is one of the relevant energy scales in this problem. And the above energy is the contribution to the hamiltonian due to the Josephson inductance.

Hamiltonian and the relevant parameters

$$H = \frac{\hat{Q}^2}{2C} - E_J cos(\frac{2\pi\phi}{\phi_0})$$
$$= \frac{2e^2}{2C}\hat{n}^2 - E_J cos(\frac{2\pi\phi}{\phi_0})$$
$$= 4E_c(\hat{n} - n_g)^2 - E_J cos(\frac{2\pi\phi}{\phi_0})$$

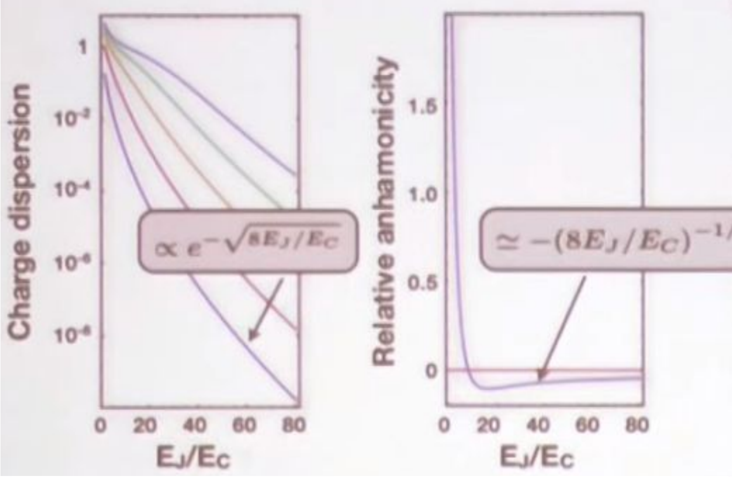


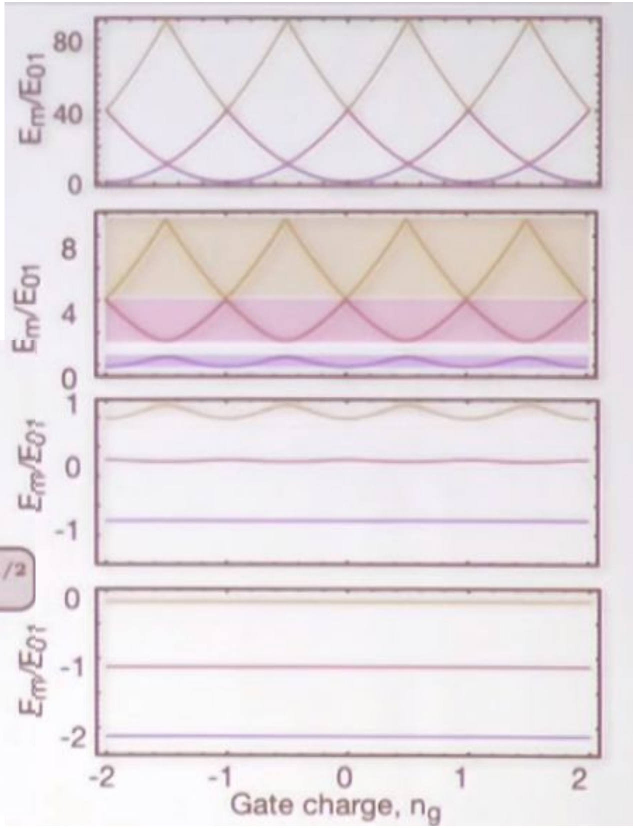
n_g can be understood as the random environmental noise which can cause decoherence.

The only other thing in our control are the two other parameters E_J, E_C .So, our goal ultimately is to make the optimal choice for the ratio of these two energies for the circuit to be minimally affected by the random noise.

Superconducting qubits: transmon region

Key words: Anharmonicity-the level spacings are a little bit off from each other(a little is enough).And that is how we make the optimal choice of $\frac{E_J}{E_C}$ to be minimally affected by random Noise.





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