Inter-qubit coupling in the circular bus architecture for superconducting qubits

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Abstract

Connectivity of qubits is an important aspect in quantum computing architectures based on superconducting qubits. The new circular bus architecture developed at the Quantum Measurement & Control lab (QuMaC) here in TIFR is a promising candidate for coupling multiple qubits more efficiently. In this report, the coupling strength between two qubits placed at different relative angles in this architecture is investigated using microwave simulations. A plot provided sheds light on the inter-qubit coupling strength for six different relative angles in the cavity excitation frequency range of 3.5-5.5 GHz. The nature of the plot resembles a theoretical prediction on the expected coupling strength well.

1 Introduction

Quantum Computers have the potential to revolutionize computing due to asymptotically faster algorithms for fundamental computational problems such as search and optimization. Among other implementations, circuit Quantum Electrodynamics (QED) with superconducting qubits is being worked on extensively due to its potential for *scalability* - which in this context is the ability to couple a large number of qubits to implement quantum algorithms.

A major experimental challenge in superconducting qubit architectures is achieving maximum connectivity of qubits as a network, when looked at as a graph. What connectivity physically means will be discussed in a later section. Suppose we have an architecture consisting of N qubits, represented by N nodes of a graph. We would ideally like a complete digraph with $\frac{N(N-1)}{2}$ directed edges to represent the geometry. This would enable the implementation of arbitrary 2 qubit gates in the network. Practically, achieving complete connectivity with sufficient coupling strength in all edges is infeasible when N is large. However there is an effort in research labs as well as industry to improve connectivity in superconducting qubit architectures. QuMaC has developed a novel circular bus cavity which can potentially couple multiple qubits more efficiently.

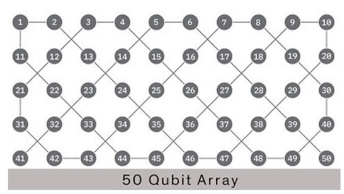

Figure 1: The connectivity diagram of IBM's 50 qubit quantum computer (Courtesy: Google images)

I worked on simulating the coupling strength between two qubits placed at different relative angles in this architecture. The work involved microwave simulations in COMSOL Multiphysics and AWR microwave office. The following section discusses the physics of superconducting qubits with an elementary mathematical treatment.

2 Superconducting Qubits and Circuit QED

Superconducting qubits are non-linear quantum LC circuits. I will motivate what this means starting from a simple lossless classical LC oscillator.

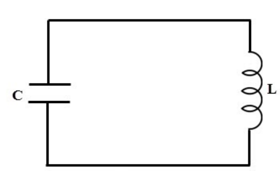

Figure 2: A simple lossless LC oscillator (Courtesy: Google Images)

The energy stored in this circuit is given by

$$E = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \tag{1}$$

where $\Phi = LI$ is the flux through the inductor. Since the energy in a lossless circuit is constant in time, $\frac{dE}{dt} = 0$, which upon simplifying yields

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q = -\Omega^2 Q \tag{2}$$

where $\Omega = \frac{1}{\sqrt{LC}}$ is the characteristic frequency of the oscillator. [Of course (2) could have also been obtained using Kirchoff's voltage rule]. Given initial conditions $Q(0)$ and $\Phi(0)$, the electrical energy stored in the circuit is constant in time and is given by $E = \frac{Q(0)^2}{2C} + \frac{\Phi(0)^2}{2L}$. Moreover, $Q(t)$ and $\Phi(t)$ are sinusoids $\frac{\pi}{2}$ out of phase. What does this result remind us of? A simple classical spring mass system!

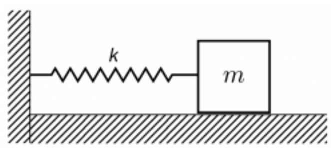

Figure 3: A simple horizontal spring-mass system (Courtesy: Google Images)

Assume a frictionless floor, and that the spring has a relaxed length of zero. The mechanical energy in the system is constant in time and is given by $E = \frac{p(0)^2}{2m} + \frac{1}{2}kx(0)^2$. $p(t)$ and $x(t)$ are sinusoids $\frac{\pi}{2}$ out of phase.

However from Quantum Mechanics, we know that position and momentum are quantum operators satisfying the canonical commutation relation $[\hat{x}, \hat{p}] = i\hbar$. The effects are Quantum Mechanics are significant in small length scales. Most first courses in Quantum Mechanics also derive the energy levels of a 1D Quantum Harmonic Oscillator (QHO) in the ladder operator formalism. In exact analogy, capacitor charge and inductor flux in circuit QED are promoted to quantum operators, satisfying $[\hat{Q}, \hat{\Phi}] = i\hbar$. We can hence write down a Hamiltonian and ladder operators for the now quantum LC oscillator as

$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} = \hbar\Omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \tag{3}$$

$$\begin{aligned} \hat{a} &= \frac{\hat{Q}}{\sqrt{2C\hbar\Omega}} + \frac{i\hat{\Phi}}{\sqrt{2L\hbar\Omega}} \\ \hat{a}^\dagger &= \frac{\hat{Q}}{\sqrt{2C\hbar\Omega}} - \frac{i\hat{\Phi}}{\sqrt{2L\hbar\Omega}} \end{aligned} \tag{4}$$

The ladder operators obey their usual commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$. The energy levels are equally spaced and are given by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\Omega \tag{5}$$

Till now we have discussed the quantum LC oscillator with a linear inductor, in the sense that the flux through the inductor is proportional to the current through it ($\Phi \propto I$), and hence the inductance L is a constant. Equally spaced energy levels are a consequence of the linearity of the circuit. However in order to fabricate a two level quantum system, we desire unequally spaced energy levels (like an atom), so that the $|0\rangle \to |1\rangle$ transition frequency (Ω_{01}) is unique as compared to other transitions. If Ω_{01} was equal to Ω_{12}, there is a chance that a qubit initially in $|1\rangle$ might transition to $|2\rangle$, which we don't want under any circumstances.

Hence came in the Josephson Junction, a dissipationless non-linear inductor, whose intricate physics is described the quantum mechanical phenomenon of superconductivity. The Josephson Junction consists of two superconductors separated a thin insulating layer. The device displays the following characteristics

$$I = I_c \sin\left(2\pi\frac{\Phi}{\Phi_0}\right) \qquad V = \frac{d\Phi}{dt} \tag{6}$$

A quantum LC oscillator with the Josephson Junction as its inductor is called a Cooper Pair Box (CPB), one the earliest models of a superconducting qubit. The hamiltonian of a CPB is weakly anharmonic (reduces to a QHO under taylor approximations), and has progressively decreasing energy level spacings. In fact, the hamiltonian of a CPB is exactly analogous to that of a quantum pendulum in a gravitational field.

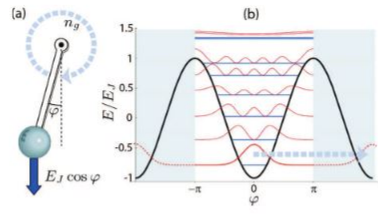

Figure 4: Energy levels of a CPB (Courtesy: Les Houches notes, S Girvin)

Since the advent of CPB, more sophisticated quantum LC circuits made with Josephson Junctions have been proposed. These include circuits like the quantronium, transmon and fluxonium. The transmon is currently the most common superconducting qubit, and is also the design used at QuMaC. The Josephson Junctions fabricated at QuMaC are typically made up of aluminium films, separated by a thin insulating layer of oxides. Each transmon is built on a rectangular silicon chip whose dimensions are a few mm.

In circuit QED, superconducting qubits are placed inside resonant cavities whose eigenmodes are in the microwave range (typically a few GHz). The cavities are then cooled down to 10-50 milli Kelvin. This makes the aluminium in the junctions superconducting, and most importantly, ensures that $k_B T \ll \hbar\Omega_{01}$, so that qubit transitions due to thermal energy are ruled out. The cavities also have ports to send and receive the GHz microwaves. Consider a simple scenario where a single qubit is placed inside a two-port cavity. A drive close to the cavity resonance frequency (ω_c) is used to measure the qubit, since the state of the qubit slightly alters the eigenfrequencies of the cavity. This is observed by a spike in reflection at resonance, and the amount of shift can be used to approximate the state of the qubit. On the other hand, drive frequencies near Ω_{01} are used to perform unitary gates.

3 Qubit-Qubit Coupling

I will now try to describe what connectivity or coupling between qubits physically means. To start with, it is instructive to consider the coupling between a single qubit and the modes of a resonant cavity. This interaction is quantum mechanical in nature, and is described by the third term of the following hamiltonian

$$H = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{\hbar\Omega_{01}}{2}\sigma_z + \hbar g(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-) + H_{drive} + H_{damping} \tag{7}$$

Popularly known as the Jaynes-Cummings hamiltonian, it has been extensively studied by the Quantum Optics community. It is the full hamiltonian describing a two-level quantum system (qubit in our case) coupled to quantized modes of the electromagnetic field (the cavity modes). The first term describes the cavity modes, the second term describes the qubit transition, and the third term represents the coupling between the qubit and the cavity modes. The last two terms, associated with damping and the electromagnetic drive, are specific to the experimental setup.

Consider the third term in the hamiltonian: $\hbar g(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-)$. Intuitively, it represents the energy associated with excitations of the qubit and cavity mediated by a single photon. g is hence a measure of the coupling strength between the qubit and cavity, also called the *vacuum rabi coupling*. In terms of lumped element electrical equivalents, g is representative of capacitive coupling between two oscillators.

The lumped element electrical equivalent of an electromagnetic system is a powerful tool to understand circuit QED. Let's say there are two qubits in a two-port cavity. Also suppose that both the excitation port as well as readout port are defined at the qubits, so a coaxial cable is effectively attached to each qubit. In this case, there are four instances of capacitive coupling in the electrical equivalent circuit. Each qubit is capacitively coupled to a coaxial cable and a two-port network (the cavity).

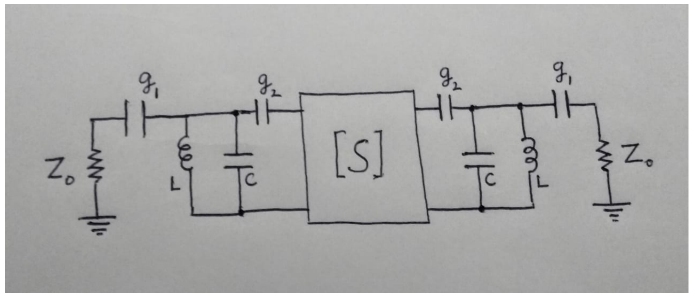

Figure 5: Circuit equivalent of 2 qubits coupled to a cavity at its ports. The $[S]$ matrix (2x2) gives the scattering parameters of the cavity

We know that coupled oscillators have characteristic normal modes. Suppose both qubits are identical with transition frequency f_{01}, and let's say the cavity has two eigenmodes of its own: one at f_a and another at f_b. Then it is observed that a drive around f_{01}, when $f_a < f_{01} < f_b$, leads to two resonances: one at $(f_{01} - g)$ and one at $(f_{01} + g)$ where $g \ll f_{01}$. Why do we see these resonances if the cavity modes are at f_a and f_b ? A simple answer is that this is a signature of *effective qubit-qubit coupling*. The observation that these peaks are symmetric about f_{01} suggests that these are normal modes of effectively coupled qubits. The word 'effective' is used in order to distinguish this from the usual capacitive coupling, which was by direct contact, whereas this coupling is via a cavity (two-port network). If one of the inductors is perturbed so as to make the qubits non-identical, the spacing between the peaks is observed to be greater than $2g$. Hence $2g$ is the minimum spacing between the normal modes if one of the qubit has a transition frequency which can be varied. This phenomenon is also referred to as an *avoided level crossing*. $2g$ is hence a measure of the qubit-qubit coupling strength. Larger $2g$ is associated with stronger coupling.

The background developed in the previous paragraph is closely related my computational work at QuMaC. The next section describes the design of the circular bus architecture, as well as aspects of my work.

4 Circular bus cavity architecture

The circular bus cavity is an novel design proposed and developed at QuMaC, with the long term goal of coupling multiple qubits more efficiently. It consists of a torus-like circular cavity with a ring-shaped aluminium core. The version already fabricated consists of four ports and chip slots at 90 degrees relative to each other. The cavity also consists of four teflon suspenders (dielectric constant 2.1) which hold the aluminium core in position. (see fig)

The cavity has two nearly degenerate eigenmodes around 3 GHz, and two similar modes around 6 GHz. The objective of my work was to optimize the design of this architecture by investigating the effective qubit-qubit coupling strength ($2g$) at relative angles of 30, 60, 90, 120, 150 and 180 degrees as the qubit transition frequency is sweeped from 3.5 to 5.5 GHz. The work involved microwave simulations of the architecture in COMSOL Multiphysics, followed by simulations of avoided level crossings in AWR Microwave office. The geometry in COMSOL looks as shown in the figure.

For each geometry, the frequency domain simulation in COMSOL is used to obtain the Scattering matrix

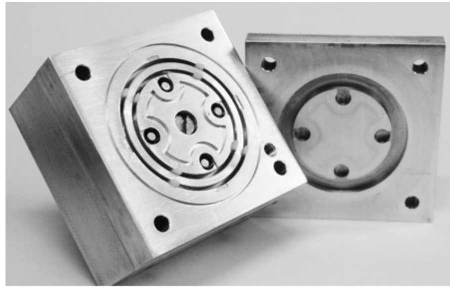

Figure 6: The circular bus cavity with 4 ports, built by QuMaC

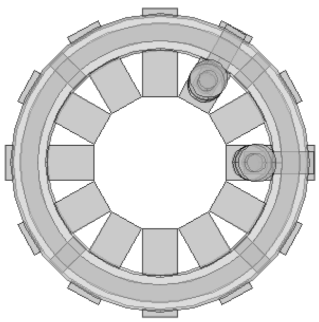

Figure 7: A sample geometry in COMSOL (60 degrees)

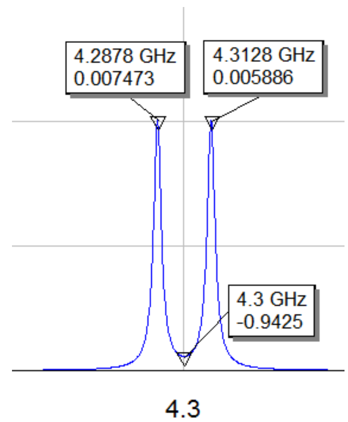

Figure 8: A sample avoided crossing in $Re(S_{11})$, as observed in AWR. The distance between the peaks gives the coupling strength $2g$

parameters of the cavity as a function of the drive frequency (3.5 to 5.5 GHz in steps of 20 MHz). This data is exported to AWR microwave office to emulate an electrical 2-port network. Equal inductors (and hence identical qubits) are connected to both ports and coupled to coaxial lines with characteristic impedences of $50k\Omega$. AWR can then simulate a measurement of $Re(S_{11})$, the real part of the reflection coefficient at port 1 for a range of excitation frequencies. This gives a bimodal curve centred at a frequency, say f. The separation between the peaks gives the coupling strength $2g$ between the qubits when both their transition frequencies are f.

5 Results

For each of the six relative angles between two qubits in the architecture, avoided crossings were noted from 3.6 to 5.4 GHz in steps of 0.1 GHz. A line plot of $2g$ versus the the qubit transition frequency was hence obtained. We find good resemblence with a theoretical prediction of the coupling strength for these angles by a PhD student at QuMaC. The theory relies on the fact that the circular bus with ports defined at the qubits can be treated as two transmission lines connected in parallel, and coupled at each end to a qubit. The relative angle between the qubits would then fix the lengths of the transmission lines. The simulations, which include exact details of the proposed structure establish confidence the theoretical calculation, which assumes an idealized model.

Several special features can be inferred from this plot in Figure 9. For $\Omega_{01} \approx 4.7GHz$, the inter-qubit coupling is equal (about 21 MHz) for relative angles of 30, 90 and 150 degrees while that for 120 degrees is nearly zero. Above 4.2 GHz, 60 and 180 degrees exhibit nearly equal coupling. 120 degrees exhibits significantly smaller coupling than other relative angles, and becomes negligibly small between between 4.5 and 4.9 GHz.

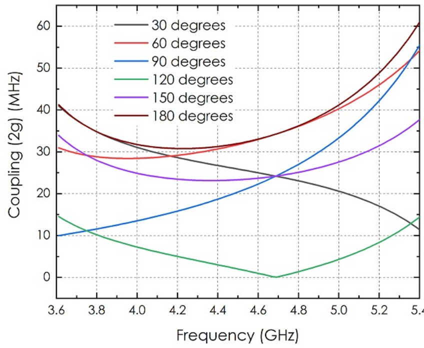

Figure 9: Theoretical plot of $2g$ vs qubit frequency for all six angles

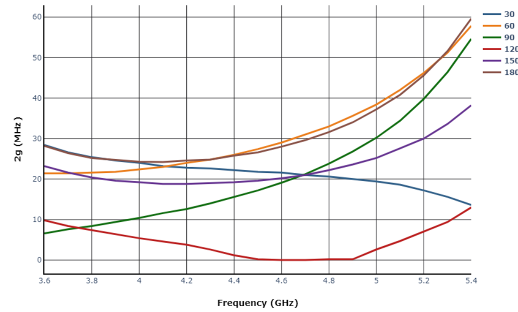

Figure 10: Simulated plot of the same using COMSOL and AWR

6 Summary & future directions

In addition to a basic introduction to the working of superconducting qubits, I touched upon the physics of qubit-qubit coupling and avoided crossings. I then described operational aspects of the simulations carried out to obtain the qubit-qubit coupling strength as a function of the qubit transition frequency in the range of 3.6 to 5.4 GHz, which matches theoretical predictions well. Among other features, the observations of minimal coupling in the architecture for qubits at a relative angle of 120 degrees, and equal coupling for three different relative angles at 4.7 GHz are interesting. For future studies, it would be exciting to see if these observations can be extrapolated when a third qubit is introduced. It is worth envisioning upto 5 qubits efficiently coupled in this architecture.

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